Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 17, pp. 3659–3666, 1996

0017–9310/96 $15.00+0.00

PII: S0017–9310(96)00029–4

Analogy between electrical machines and heat transfer-irreversible heat engines

A. BEJAN and N. DAN

Department of Mechanical Engineering and Materials Science, Duke University, Durham, NC 27708-0300, U.S.A.

(*Received* 22 *June* 1995 *and in final form* 28 *December* 1995)

Abstract—This paper extends to electrical machines the thermodynamics and heat transfer optimization approach that has been developed for heat engines. Four models of thermodynamically irreversible electrical motors with heat transfer to the ambient are proposed and optimized: general reversible motor in series with its resistance, induction motor with transformer, synchronous motor with transformer and direct current motor. The conversion efficiency at maximum power is 1/2. When, as in specific applications, the operating temperature of the windings must not exceed a specified level, the power output is lower and the efficiency higher. In the case of an electrical power generator it is shown that the generator and the heat engine that drives it can be optimized separately for maximum power. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

A noteworthy trend in modern heat transfer is the use of thermodynamic principles in close combination with heat and mass transfer and fluid mechanics principles [1]. The objective of such work is to develop fundamental models and optimization results (trade-offs, extrema) for real (irreversible) devices and processes. This growing body of literature is extremely diverse [1]: recent applications include the optimization of sensible-heat storage [2], heat transfer augmentation techniques [3], mixed convection in a vertical duct with baffles [4], and the burning of a droplet in a stream [5]. This field was reviewed on several occasions (e.g. refs. [1, 6–8]): an important part of this work refers to the modeling of irreversible heat engines and refrigerators, with the objective of determining the operating conditions for maximum power output from heat engines, and minimum power input for refrigerators.

Heat engines and refrigerators are representative of the class of thermomechanical power converters. On this background, in this paper we extend fundamentally the modeling method and optimization procedure to electromechanical power converters such as electrical motors and power generators. Electromechanical devices transform one form of work into another (conversion of energy from electrical into mechanical, or *vice-versa*). Two energy transformations are involved: between the electrical circuit and the magnetic field, and between the magnetic field and the mechanical system. The conversion is partial because of the intrinsic thermodynamic irreversibility of the electrical circuits that are built into the machine. The lone heat interaction occurs between the machine (e.g. armature windings and ferromagnetic core) and the ambient. The purpose of this interaction is to reject to the atmosphere the portion of the input power that is destroyed through the irreversible operation of the machine.

The single temperature reservoir and single heat transfer interaction distinguish electromechanical converters from the heat engines and refrigerators that have been studied in the past. Nevertheless, as we begin to show in the next section, a most basic analogy exists between the maximum power conditions of electromechanical and thermomechanical conversion systems. The analogy begins with the observation that the two electric potentials of an electric motor play roles similar to the temperature reservoirs of a heat engine (note: temperature reservoirs, not 'heat' reservoirs: see also the concluding paragraph of the paper).

ELECTROMECHANICAL VS THERMOMECHANICAL CONVERTERS

The simplest way to illustrate the analogy between the maximum power conditions of electromechanical converters and thermomechanical converters (heat engines) is by analyzing the electrical circuit shown on the left side of Fig. 1. The power source delivers the current I at the voltage V_1. A reversible electrical motor receives power at the voltage V_2 through a constant electrical resistance R, separated from the conservative part of the machine. The ground voltage is V_0.

The question we address is how to maximize the power output of the motor, namely

$$\dot{W} = V_1 I - RI^2 - V_0 I. \quad (1)$$

The terms on the right side represent, in order, the power drawn from the V_1 source, the power destroyed

NOMENCLATURE

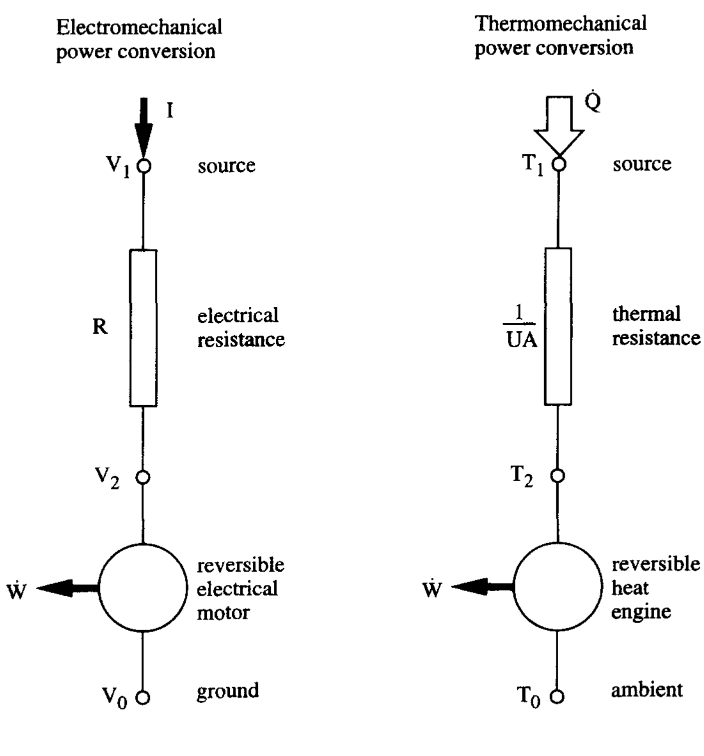
a	number of armature circuits in parallel
A	heat transfer area
E	back electromotive force
I	current
k_m, k_t	transformer ratios of motor and transformer
n	rotational speed
N	number of winding conductors
p	number of pole pairs
R	resistance
T	temperature
U	overall heat transfer coefficient
U	voltage difference
V	voltage
$\dot{W}$	power
X	leakage reactance.

Greek symbols

ΔU	brush contact voltage
η	efficiency
Φ	magnetic flux per pole.

Subscripts

a	armature
cm	core of motor
e	effective
f	friction
max	maximum
opt	optimum
s	shaft, synchronous.



$$\eta = \frac{1}{2}\left(1 - \frac{V_0}{V_1}\right) \qquad \eta = 1 - \left(\frac{T_0}{T_1}\right)^{1/2}$$

$$(V_2 - V_0)_{opt} = \frac{1}{2}(V_1 - V_0) \qquad \left(\frac{T_2}{T_0}\right)_{opt} = \left(\frac{T_1}{T_0}\right)^{1/2}$$

Fig. 1. Simple model of an electrical motor with its resistance in series, and the analogy between the maximum-power operation of electromechanical and thermomechanical power converters.

by Joule heating in the line resistance, and the electrical power that enters the ground node. The core losses and mechanical friction losses are neglected. Equation (1) shows that the power output $\dot{W}$ is zero in two extremes, at zero current ($I = 0$) and at zero voltage difference $[V_2 = V_0, I = (V_1 - V_0)/R]$. The maximum power,

$$\dot{W}_{max} = \frac{(V_1 - V_0)^2}{4R} \quad (2)$$

occurs at the intermediate current

$$I_{opt} = \frac{V_1 - V_0}{2R}. \quad (3)$$

Under the same conditions, the voltage difference across the motor is exactly half of the voltage difference between source and ground,

$$(V_2 - V_0)_{opt} = \tfrac{1}{2}(V_1 - V_0). \quad (4)$$

The efficiency of converting the power drawn from the source into maximum mechanical power output, $\eta = \dot{W}_{max}/(V_1 I_{opt})$, is

$$\eta = \frac{1}{2}\left(1 - \frac{V_0}{V_1}\right). \quad (5)$$

The maximum power conditions (2)–(5) hold for d.c. and a.c. circuits as well. The heat engine analog of the electrical system optimized above is shown on the right side of Fig. 1. A reversible heat engine receives heat from the temperature T_1 through a heat exchanger of area A and overall heat transfer coefficient U. The cold end of the heat engine is in equilibrium with the ambient T_0. The maximum power conditions of the heat engine with thermal resistance are well known [1, 8–11]: listed in Fig. 1 are the conversion efficiency $\eta = 1 - (T_0/T_1)^{1/2}$ and the ratio of the temperatures that sandwich the heat engine. By comparing the two sides of Fig. 1, we see at one glance the analogy between maximum power output in electromechanical and thermomechanical converters. It is interesting that the ratio 1/2 appears as an exponent in the heat engine formulas, and as a factor in the electrical motor formulas. It can be shown that the efficiency listed in equation (5) does not change if an electrical resistance is added between the motor and the ground: this finding is analogous to the maximum power efficiency formula of heat engines with hot-end and cold-end thermal resistances.

In the next five sections we develop the maximum power conditions for several classes of the most common electrical machines that are in use today. To simplify the analysis we use the electrical engineering convention of writing zero for the ground voltage, instead of V_0.

INDUCTION MOTOR WITH TRANSFORMER

Figure 2 shows an induction motor connected to the three-phase a.c. power supply through a transformer. The power drawn from the network is characterized by V and I, which are the effective voltage and current per phase. The transformer ratio k_t is given, $k_t = V/U_1$. In this definition, U_1 is the voltage across the terminals of the secondary winding of the transformer, or of the stator of the induction motor. The current in the stator is $I_1 = I$. The transformer ratio k_m of the motor is also known, $k_m = U_1/U_2$. If we neglect the core losses in the motor, the transversal part of the electrical scheme of Fig. 2 vanishes, the magnetizing current becomes zero, and

$$k_m \cong \frac{I_2}{I_2'} \cong \frac{I_2}{I_1}, \quad (6)$$

where I_2' and I_2 are the rotor phase current referenced to the motor primary and the real phase current. We also assume that the rotational speed of the shaft does not vary appreciably. Consequently the power factor for this type of motor is small ($\cos\varphi \approx 0.7\ldots0.8$) and does not change significantly as the current varies and the terminal voltage is kept constant.

The equivalent electrical circuit of the transformer and motor combination is shown in the lower part of Fig. 2. All the parameters indicated on this circuit have been referenced to the transformer primary through the relations

$$X_{2t}' = k_t^2 X_{2t} \quad R_{2t}' = k_t^2 R_{2t} \quad (7)$$

$$X_{1m}' = k_t^2 X_{1m} \quad R_{1m}' = k_t^2 R_{1m} \quad R_{2m}' = k_t^2 k_m^2 R_{2m}. \quad (8)$$

Parameters X_{2t}, R_{2t}, X_{1m}, R_{1m} and R_{2m} represent, in order, the transformer secondary leakage reactance, transformer secondary resistance, motor primary leakage reactance, motor primary resistance and motor secondary resistance.

Finally, the equivalent circuit of Fig. 2 shows that the power output of the motor can be written as

$$\dot{W} = 3VI\cos\varphi - 3R_e I^2 - \dot{W}_f. \quad (9)$$

On the right side R_e is the equivalent resistance $R_e = R_{1t} + k_t^2(R_{2t} + R_{1m} + k_m^2 R_{2m})$, and $\dot{W}_f$ is the mechanical power destroyed by shaft friction. As noted earlier, we treat $\cos\varphi$ and $\dot{W}_f$ as independent of I, and maximize $\dot{W}$ with respect to I; the results are

$$\dot{W}_{max} = \frac{3V^2}{4R_e}\cos^2\varphi - \dot{W}_f \quad I_{opt} = \frac{V}{2R_e}\cos\varphi. \quad (10)$$

If the transformer is absent and the induction motor is optimized alone, then equations (10) continue to apply, provided $R_e = R_{1m} + k_m^2 R_{2m}$. The corresponding efficiency ratio $\eta = \dot{W}_{max}/(3VI_{opt}\cos\varphi)$ is essentially equal to 1/2,

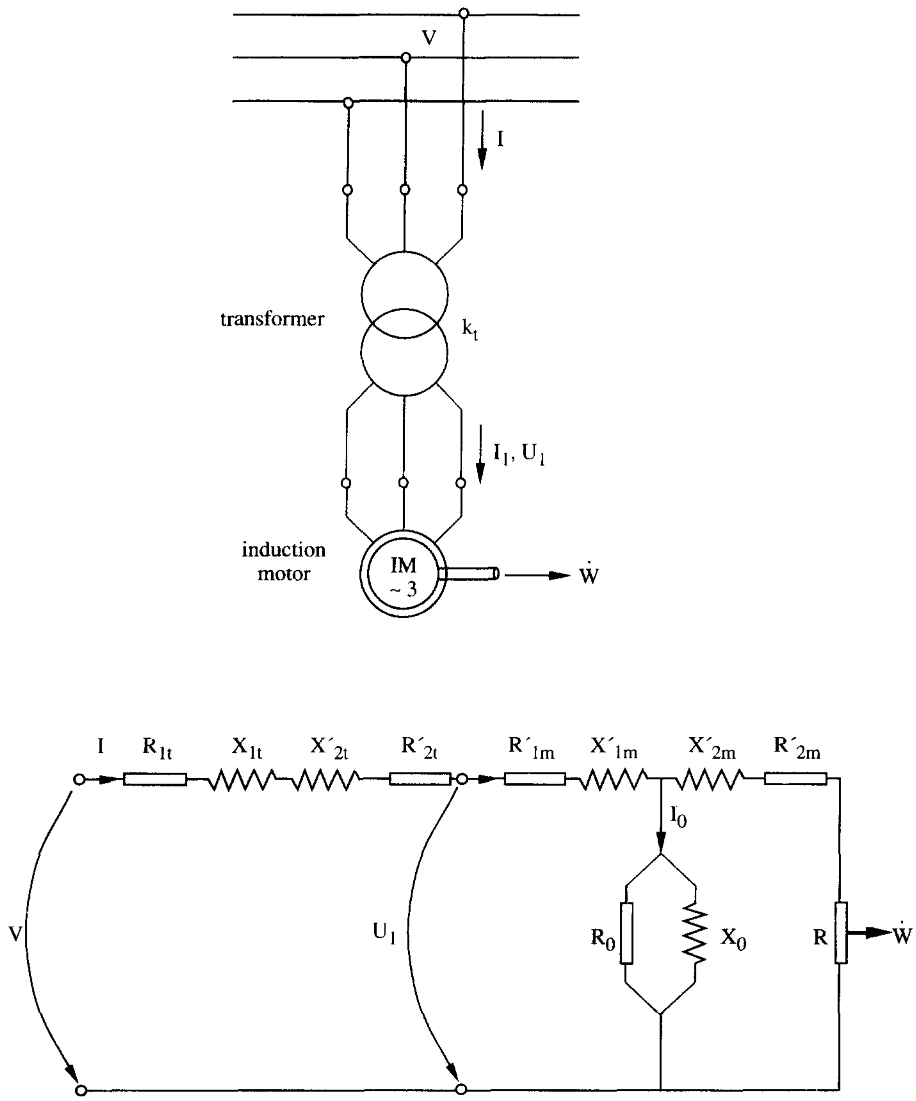


Fig. 2. Induction motor with transformer, and the equivalent electrical circuit of the combined system.

$$\eta = \frac{1}{2} - \frac{\dot{W}_f}{(3V^2/2R_e)\cos^2\varphi} \cong \frac{1}{2} \tag{11}$$

because the power destroyed by shaft friction is expected to be much smaller than the power drawn from the source.

The maximum power output (10) increases as the equivalent-circuit resistance R_e decreases. This feature is analogous to the dependence between the maximum power of a power plant and the total thermal resistance of the heat exchangers (e.g. ref. [8, p. 410]). It is also worth noting that traditionally, induction motors are designed at operating currents significantly lower than I_{opt} of equation (10) [12], which means that the power output $\dot{W}$ and the associated Joule heating losses are much lower than in the maximum-power design equations (10) and (11). This is why the electromechanical power conversion efficiency of traditional designs is higher ($\eta \sim 0.8 \ldots 0.95$, depending on power) than the maximum-power efficiency $\eta \cong 1/2$ determined in equation (11).

SYNCHRONOUS MOTOR WITH TRANSFORMER

The maximum-power operating conditions of other types of electric machines can be determined by following the method used for induction motors. Figure 3 shows the conventional representation of a synchronous motor connected through a transformer to the three-phase power network. Several of the modeling features introduced in the preceding section and Fig. 2 continue to apply. The secondary parameters (R_2', X_2') are referenced to the transformer primary: $R_2' = k_t^2 R_2$ and $X_2' = k_t^2 X_2$. The current in the equivalent transformer secondary is almost equal to the current in the primary, $I_2' \cong I$, which means that the magnetization current and core losses are negligible [12].

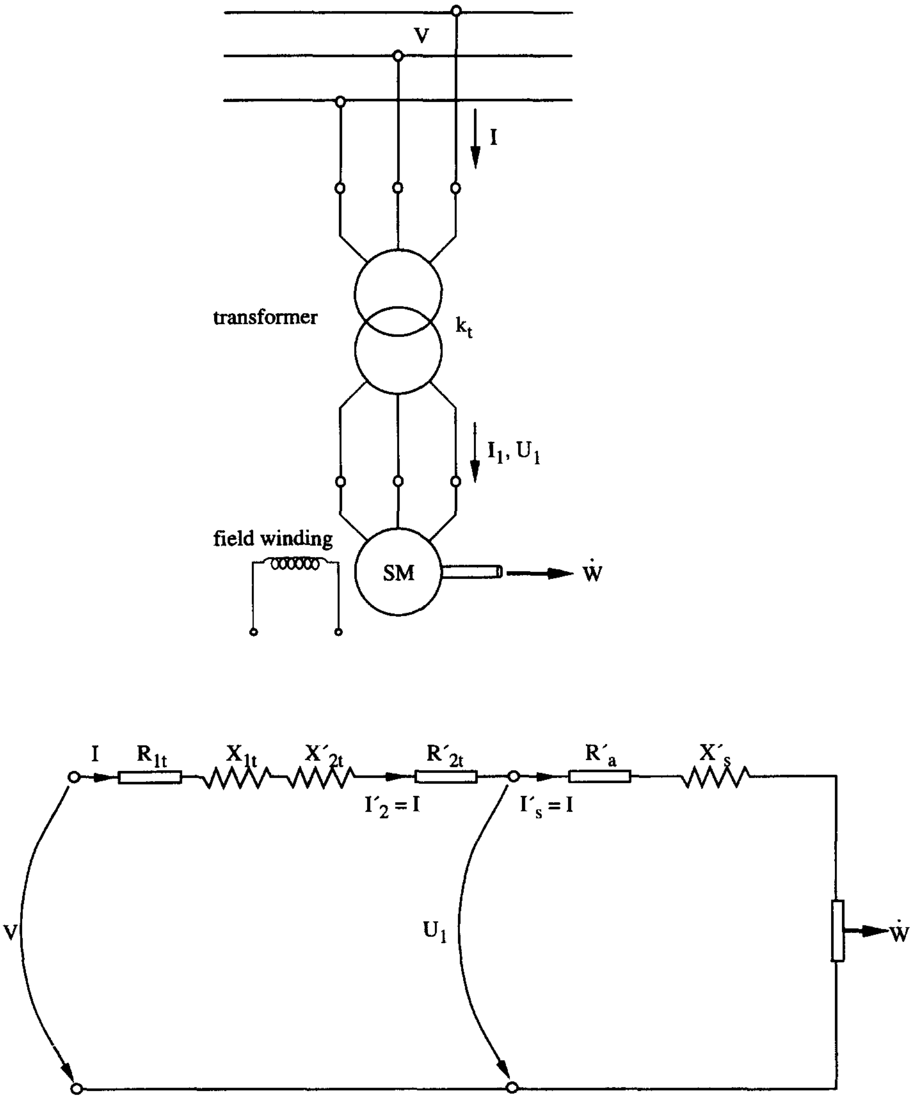


Fig. 3. Synchronous motor with transformer, and the equivalent electrical circuit of the combined system.

The lower part of Fig. 3 shows the equivalent circuit of the system composed of a nonsalient (uniform air-gap) synchoronous motor and transformer. In order to keep the same current I in the synchoronous motor part of the circuit (namely $I_s' = I$), the armature resistance and synchronous reactance per phase are referenced to the transformer primary: $R_a' = k_t^2 R_a$ and $X_s' = k_t X_s$. In these relations R_a and X_s are the armature resistance per phase and the synchronous reactance per phase. Finally, it is reasonable to assume that the power factor of the synchronous motor is approximately equal to 1 and, consequently, the power factor of the transformer has the same value.

The total power drawn from the network is $3VI\cos\varphi \cong 3VI$, where V and I are the effective voltage and current in the transformer primary. According to the equivalent circuit of Fig. 3, the mechanical power output of the motor is

$$\dot{W} = 3VI - 3R_e I^2 - \dot{W}_f - \dot{W}_{cm}, \qquad (12)$$

where R_e is the equivalent resistance of the system, $R_e = R_1 + k_t^2(R_2 + R_a)$, and $\dot{W}_f$ and $\dot{W}_{cm}$ are the mechanical power loss due to shaft friction and air drag, and the electromagnetic power loss due to eddy currents in the core of the motor. Solving $\partial\dot{W}/\partial I = 0$, we obtain the following maximum power output and optimal current:

$$\dot{W}_{max} = \frac{3V^2}{4R_e} - \dot{W}_f - \dot{W}_{cm}, \quad I_{opt} = \frac{V}{2R_e}. \qquad (13)$$

Once again, the electromechanical conversion efficiency at maximum power ($\eta = \dot{W}_{max}/(3VI)$) is close to 1/2, because the friction and core losses are small in comparison with the power drawn from the network,

$$\eta = \frac{1}{2} - \frac{\dot{W}_f + \dot{W}_{cm}}{3V^2/(2R_e)} \cong \frac{1}{2}. \qquad (14)$$

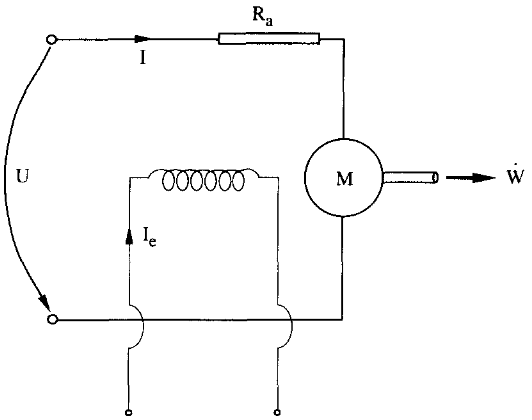


Fig. 4. The equivalent electrical circuit of a direct current motor.

The observations made at the end of the section on induction motors also apply to the maximum-power operation of synchronous motors, and are not repeated here. Finally, if the transformer is not present in the system of Fig. 3, the maximum power operation is represented by equations (13) and (14) with $R_e = R_a$.

DIRECT CURRENT MOTOR

The steady-state operation of a separately excited d.c. motor is described [12] by the equivalent circuit of Fig. 4 and

$$U = E + R_a I + \Delta U \tag{15}$$

with $E = k\Phi n$ and $k = (p/a)$, where E, U, R_a and I are the back electromotive force, the terminal d.c. voltage, the resistance of the rotor armature windings, and the armature current. The brush contact voltage drop ΔU is small (~ 2 V) and will be neglected. In these equations, Φ, n, p, N and a are the magnetic flux per pole, the rotational speed, the number of pole pairs, the number of winding conductors, and the number of armature circuits in parallel. The mechanical power output of the motor is

$$\dot{W} = UI - R_a I^2 - \dot{W}_f - \dot{W}_{cm}, \tag{16}$$

where $\dot{W}_f$ and $\dot{W}_{cm}$ are the friction and rotor core losses. In the maximization of $\dot{W}$ with respect to I we treat $(\dot{W}_f + \dot{W}_{cm})$ as a constant, and obtain,

$$\dot{W}_{max} = \frac{U^2}{4R_a} - \dot{W}_f - \dot{W}_{cm}, \quad I_{opt} = \frac{U}{2R_a} \tag{17}$$

$$\eta = \frac{\dot{W}_{max}}{UI} = \frac{1}{2} - \frac{\dot{W}_f + \dot{W}_{cm}}{U^2/(2R_a)} \cong \frac{1}{2}. \tag{18}$$

The optimization of d.c. motors (and any of the motor types considered earlier) can be pursued based on models that are more refined than the ones used in this paper. For example, the assumption that the frictional power loss $\dot{W}_f$ is a constant independent of I can be relaxed by noting that, from equation (15), the rotational speed decreases slowly as I increases, $n = (U - R_a I)/k\Phi$. Consequently, in the improved model $\dot{W}_f$ may be written as the sum of two terms, one proportional to n^2, accounting for the laminar shear flow of the shaft lubricant, and the other proportional to n^b (where $2 < b < 3$) to account for the turbulent flow in the air gap and between other parts of the housing.

The observations discussed at the end of the section on induction motors also apply to d.c. motors. For example, the optimal current (17) refers to operation well above the temperature limit and admissible armature resistance of actual d.c. motor designs. Consider as a numerical example the rated values of a commercial d.c. motor: $U = 110$ V, $I = 30$ A, $R_a = 0.25$ Ω, $n = 1500$ rev/min, and $\dot{W} = 2700$ W. The real efficiency of this motor is $\eta_r = \dot{W}/(UI) = 2700$ W/3300 W $= 0.81$. Its total power loss (3300 − 2700 W = 600 W) is split between $I^2 R_a = 225$ W and $(\dot{W}_f + \dot{W}_{cm}) = 375$ W. Turning our attention now to equations (17) and (18), we find that the optimal current ($I_{opt} = 220$ A) is more than seven times greater than the rated current. The maximum power would be $\dot{W}_{max} = 11\,725$ W, with the corresponding efficiency $\eta = 0.48$. One practical aspect of the maximum-power result (17) is that to increase the power output of a d.c. machine one must decrease as much as possible its resistance, R_a. It is worth keeping in mind that in practical applications subjected to a maximum operating temperature, the maximum efficiency and power output are registered when the constant losses (core losses) are equal to the variable losses (Joule heating) [12].

OPERATION SUBJECT TO TEMPERATURE CONSTRAINT

The point made numerically in the above example can be illustrated in general terms by reconsidering the simple model of an irreversible electrical motor, namely the reversible motor with series resistance (Fig. 1, left side). The Joule heating generated in the resistance R is transferred to the ambient of temperature T_0, namely $I^2 R = UA(T - T_0)$. In this expression T is the operating temperature of the resistance, and UA is the assumed constant thermal conductance between the resistance (i.e. windings) and ambient air. If the windings are such that T_{max} is the temperature that cannot be exceeded by the operating temperature T, then the $I^2 R$ power loss must satisfy the condition,

$$I^2 R \leqslant UA(T_{max} - T_0) \tag{19}$$

Equations (1)–(3) and $V_0 = 0$ can be rearranged to express in dimensionless form the operation under conditions other than maximum power,

$$\frac{\dot{W}}{\dot{W}_{max}} = 2\frac{I}{I_{opt}} - \left(\frac{I}{I_{opt}}\right)^2, \quad \eta = 1 - \frac{I}{2I_{opt}}. \tag{20}$$

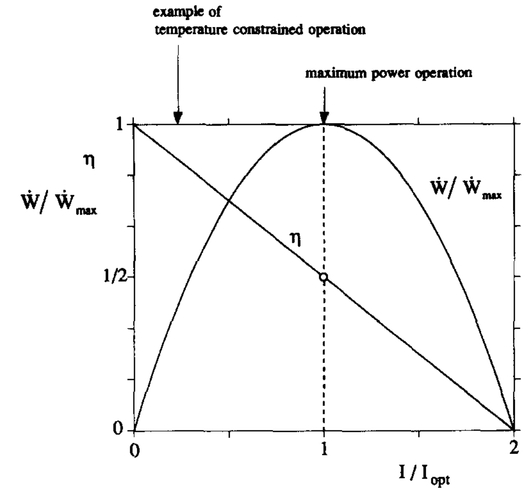


Fig. 5. The power output and efficiency of the simple model of Fig. 1 (left side) under conditions other than maximum power output.

These two expressions are plotted in Fig. 5, and show once again that maximum power corresponds to $\eta = 1/2$. Expressed in terms of the same dimensionless abscissa parameters (I/I_{opt}), the maximum temperature condition (19) becomes

$$\frac{I}{I_{opt}} \leqslant \left[\frac{4R}{V_1^2}UA(T_{max}-T_0)\right]^{1/2}. \quad (21)$$

As shown in the numerical example that ended the section on d.c. motors, the group on the right side can be much smaller than 1: such temperature constrained designs fall to the left of $I/I_{opt} = 1$ in Fig. 5. The same conclusion can be drawn analytically by combining equations (20) and (21)

$$\eta \geqslant 1-\frac{1}{2}\left[\frac{4R}{V_1^2}UA(T_{max}-T_0)\right]^{1/2} > \frac{1}{2}. \quad (22)$$

In conclusion, maximum power operation at maximum temperature is achieved on the left side of the $\dot{W}$ peak (Fig. 5), at the current I calculated based on equation (21) with equal sign. The more stringent the temperature constraint, the lower the power output and the higher the efficiency.

ELECTRICAL POWER GENERATOR

Our main objective in this paper was to extend to electric machines the maximum-power optimization method that was developed for heat engines. The four models analyzed until now were all motors, i.e. converters of electrical work into mechanical work. In this section we consider the reverse application, namely a generator of electrical power. The model is shown in Fig. 6: the actual generator is composed of a reversible generator in series with its own resistance (e.g. armature resistance). The electrical power generator is driven by the mechanical power delivered by an irreversible heat engine operating between T_1 and T_0. The heat engine can be modeled in several ways [1], by accounting for the major irreversibility mechanisms of the engine. The key question is whether the generator and the heat engine should be optimized as an ensemble, or separately. Let $\dot{W}_s$ be the shaft power received by the generator from the heat engine, $\dot{W}_s = V_2 I$. The electrical power output, $\dot{W}_e = V_1 I$, can also be written as

$$\dot{W}_e = \dot{W}_s - R\left(\frac{\dot{W}_s}{V_2}\right)^2. \quad (23)$$

First, we may regard V_2 as fixed and maximize $\dot{W}_e$ with respect to $\dot{W}_s$ (or I): the results,

$$\dot{W}_{s,opt} = \frac{V_2^2}{2R}, \quad \dot{W}_{e,max} = \tfrac{1}{2}\dot{W}_{s,opt} \quad (24)$$

show that the conversion efficiency is 1/2, and that the maximum electrical power output is proportional to the mechanical power delivered by the heat engine. This means that for maximum electrical power output the heat engine too must be designed for maximum power, and that the heat engine optimization can be performed *separately*. In other words, all the maximum power results that have been developed for heat engines [1] are relevant to the maximization of power output from electrical generators.

Second, we may regard the output voltage V_1 as fixed, and rewrite $\dot{W}_s = I^2R + \dot{W}_e$ as

$$\dot{W}_s = \left(\frac{\dot{W}_e}{V_1}\right)^2 R + \dot{W}_e. \quad (25)$$

Equation (24) shows that the electrical power output varies monotonically with the shaft power input. The preceding conclusions continue to apply: maximum electrical power output demands maximum mechanical power from the heat engine, the generator and the engine can be optimized separately, and the results developed for heat engines can be used for electrical power generators driven by heat engines.

CONCLUSION

In this paper we have established a direct analogy between the optimization of irreversible heat engines and the optimization of electrical motors and power generators. We showed that maximum mechanical power output is achieved when the electromechanical conversion efficiency is approximately equal to 1/2. This conclusion is independent of the motor type: we reached it using the two-component model of Fig. 1 (left side), the induction motor and transformer model of Fig. 2, the synchronous motor and transformer model of Fig. 3 and the direct current motor model of

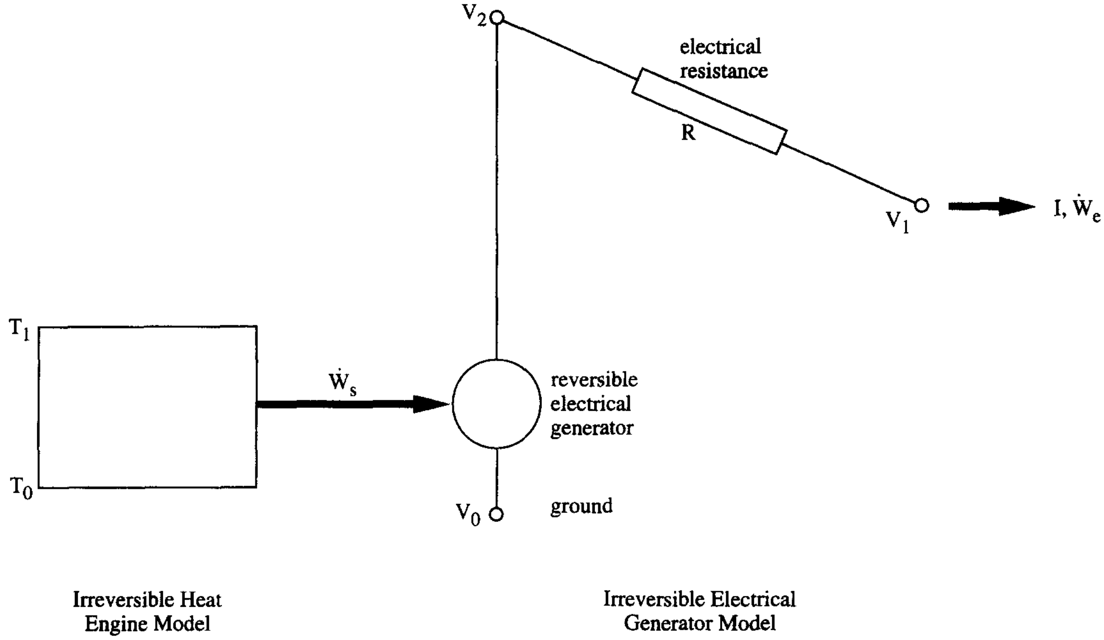


Fig. 6. Model of an irreversible electrical power generator driven by an irreversible heat engine.

Fig. 4. Present motor designs are constrained by the maximum operating temperature of the windings. Their power output is lower than the maximum, and the efficiency is higher than 1/2. Electrical power generators can also be optimized for maximum power output. This can be accomplished by optimizing separately the electrical power generator and the heat engine that drives it.

In summary, we have extended to electrical machines the fundamental modeling and optimization method based on combined thermodynamics and heat transfer [1, 6]. An unexpected payoff from the analogy constructed in this paper is the observation that the well-known heat engine model (Fig. 1, right side) is *incomplete* because it does not show the system (e.g. hardware) that delivers the freely-varying heat input $\dot{Q}$. It is true that on the left side of Fig. 1, the current I can be *varied* during the maximization of $\dot{W}$ because it is drawn from an electric power network assumed *infinite* in its capacity of serving as a current supply. There is no such network (infinite, high-temperature "heat" supply) to which we might connect the top end of the right side of Fig. 1. A complete thermodynamic optimization of the heat engine model must take into account the finite resources (e.g. fuel) that are responsible for the assumed variable heat input. This important and usually overlooked aspect of heat engine thermodynamics is discussed further in Refs [1, pp. 227–232] and Ref. [13].

Acknowledgements—This work was supported by the National Science Foundation and Duke University.

REFERENCES

1. A. Bejan, *Entropy Generation Minimization*. CRC Press, Boca Raton, FL (1996).
2. R. J. Krane, A second law analysis of the optimum design and operation of a thermal energy storage system, *Int. J. Heat Mass Transfer* **30,** 43–57 (1987).
3. R. C. Prasad and J. Shen, Performance evaluation of convective heat transfer enhancement devices using exergy analysis, *Int. J. Heat Mass Transfer* **36,** 4193–4197 (1993).
4. C. H. Cheng, W. P. Ma and W. H. Huang, Numerical predictions of entropy generation for mixed convective flows in a vertical channel with transverse fin arrays, *Int. Commun. Heat Mass Transfer* **21,** 519–530 (1994).
5. I. K. Puri, Second law analysis of convective droplet burning, *Int. J. Heat Mass Transfer* **35,** 2571–2578 (1992).
6. A. Bejan, *Entropy Generation through Heat and Fluid Flow*. Wiley, New York (1982).
7. M. Feidt, *Thermodynamique et Optimisation Energetique des Systems et Procedes*. Technique et Documentation, Lavoisier, Paris (1987).
8. A. Bejan, *Advanced Engineering Thermodynamics*. Wiley, New York (1988).
9. P. Chambadal, *Les Centrales Nucleaires*, pp. 41–58. Armand Colin, Paris (1957).
10. I. I. Novikov, The efficiency of atomic power stations, *J. Nucl. Energy II* **7,** 125–128 (1958), translated from *Atomnaya Energiya* 3, 409 (1957).
11. F. L. Curzon and B. Ahlborn, Efficiency of a Carnot engine at maximum power output, *Am. J. Phys.* **43,** 22–24 (1975).
12. S. J. Chapman, *Electric Machinery Fundamentals*. McGraw-Hill, New York (1985).
13. A. Bejan, Models of power plants that generate minimum entropy while operating at maximum power, *Am. J. Phys.* **64** (in press).